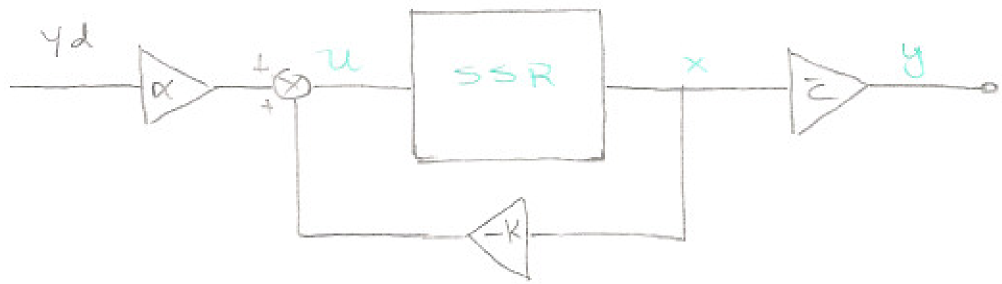
SET POINT REGULATION

OBJECTIVE:

Stable closed loop system (& desired performance)

$$\lim_{t \to \infty} y(t) \longrightarrow y_d$$

set point



$$\dot{x} = Ax + Bu$$

$$y = Cx$$

let $\quad u = -Kx + \alpha y_d$

$$\dot{x} = Ax + B[-Kx + \alpha y_d]$$

$$\dot{x} = (A - BK)x + B\alpha y_d \qquad (1)$$

note: design K such that the eigen values of (A − BK) are in LHP

note: Design α such that

$$\lim_{t \to \infty} y(t) = y_d$$

apply $\mathcal{L}$ to ①

$$sX(s) = (A - BK)X(s) + B\alpha\, y_d(s)$$

$$[sI - (A - BK)]\,X(s) = B\alpha\, y_d(s)$$

$$X(s) = [sI - (A - BK)]^{-1} B\alpha\, y_d(s)$$

$$y(s) = CX(s) \Rightarrow C[sI - (A - BK)]^{-1} B\alpha\, y_d(s)$$

Final value theorem.

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,y(s) = s\,C(sI - (A - BK))^{-1} B\alpha\, y_d(s)$$

y_d: step input

$$y_d = \frac{1}{s}$$

$$\Rightarrow \lim_{t\to\infty} y(t) = \lim_{s\to 0} \cancel{s}\, C(sI - (A - BK))^{-1} B\alpha \frac{1}{\cancel{s}}$$

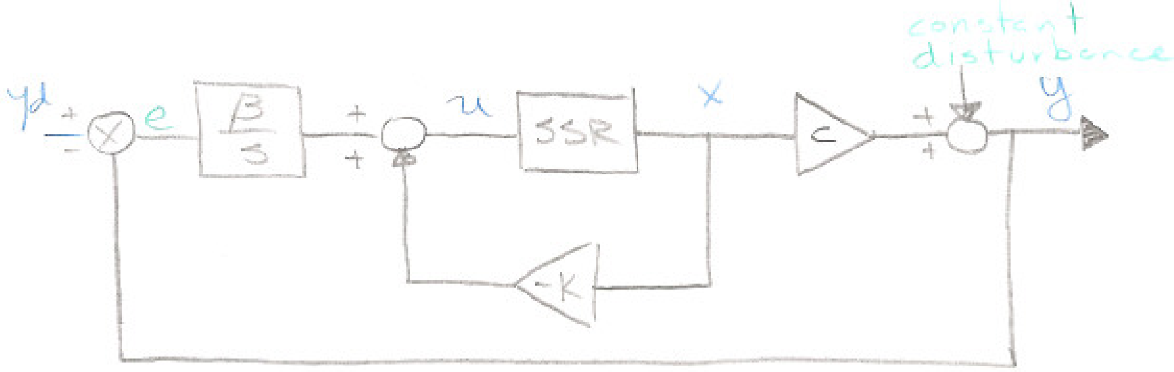
$$= \underbrace{C(BK - A)^{-1} B}_{\text{scalar}}\alpha = 1$$

we want 1

$$\therefore \quad \alpha = \frac{1}{C(BK - A)^{-1} B}$$

we do not design controllers like this.

More Practical Set Point Regulation



note: Assuming y_d and disturbance are constant.

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

$$u = -Kx + \beta \int_0^t \underbrace{(y_d - y(\tau))}_{e} \, d\tau$$

Design K such that the closed loop system is stable (& specified performance)

$$u = -Kx + \beta\alpha$$

where

$$\alpha = \int_0^t (y_d - y(\tau))\, d\tau$$

$$\dot{\alpha} = y_d - y(t) = y_d - Cx$$

hence, system becomes

$$\dot{x} = Ax + Bu$$
$$\dot{\alpha} = -Cx + y_d$$

$$u = -Kx + \beta\sigma$$

$$z = \begin{bmatrix} x \\ \sigma \end{bmatrix}, \quad z \in \mathbb{R}^{n+1}$$

$$u = -\bar{K} z, \quad \bar{K} = [K, -\beta]$$

rewrite the SSR.

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{\sigma} \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} A & \overset{n\times 1}{\boxed{0}} \\ -C & \underset{1\times 1}{\boxed{0}} \end{bmatrix}}_{A^*} \underbrace{\begin{bmatrix} x \\ \sigma \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B^*} u + \begin{bmatrix} 0 \\ y_d \end{bmatrix}$$

the overall closed loop system becomes.

$$\dot{z} = (A^* - B^*\bar{K})z + \begin{bmatrix} 0 \\ y_d \end{bmatrix}$$

Design $\bar{K}$ such that the eigen values of $(A^* - B^*\bar{K})$ are in the left hand plane according to specified performance.

note: A^* is a $(n*1)\times(n+1)$ matrix.